STABILITY OF BOUNDARY LAYER OF CATALYTICALLY RECOMBINING GAS

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UDC 532.526

A study is made of the stability of the boundary layer of a partially dissociated diatomic gas. The surface over which flow occurs is catalytically active and the gas recombines on it. Analysis of stability is performed in the Dunn and Lin approximation [1]. The equations for small perturbations in the shape of a traveling wave have the form

$$i(u-C)r+\rho'\varphi+\rho(if+\varphi')=0;$$
$$\rho[i(u-C)\gamma+c'\varphi]=\mu\gamma''/\alpha\,\mathrm{ReSc};$$
$$\rho\alpha^2(u-C)\varphi=i\pi'/\varkappa_\infty M_\infty^2;$$
$$\rho[i(u-C)f+u'\varphi]=-i\pi/\varkappa_\infty M_\infty^2+\mu f''/\alpha\,\mathrm{Re};$$
$$\rho[i(u-C)\vartheta+T'\varphi]=i(u-C)\pi(c_{p\infty}-c_{v\infty})/c_p+\mu\vartheta''/\alpha\,\mathrm{RePr};$$
$$\pi=\gamma/(1+c)+r/\rho+\vartheta/T,$$

where u is the gas flow velocity in the boundary layer; ρ is the density; T is the temperature; c is the degree of dissociation; μ is viscosity; c_p and c_v are the specific heat capacities at constant pressure and volume; $\varkappa_\infty = c_{p\infty}/c_{v\infty}$; M_∞ is the Mach number at the outer edge of the boundary layer; Pr is the Prandtl number; Sc is the Schmidt number; Re is the Reynolds number; α is the wave number; C is the phase velocity for perturbation propagation; f, φ, π, r, ϑ, and γ are amplitude functions of the pulsations of the quantities u, v/α, p, ρ, T, and c, respectively; p is the pressure; v is the transverse component of the velocity. Primes denote differentiation with respect to the transverse coordinate y and the subscript ∞ denotes values of quantities at the outer edge of the boundary layer, along which some of the quantities are scaled. Thermal diffusion is neglected.

Boundary conditions at the outer edge of the boundary layer are determined from the conditions for perturbation damping outside the boundary layer. To the boundary conditions for velocity pulsations at the surface ($f_0 = \varphi_0 = 0$; the subscript 0 refers to values at the wall), must be added two conditions imposed on pulsations of state parameters. One of them is determined by the requirement for continuity in the values of pulsations of temperature and of the normal component of the thermal flux during passage through the gas–material–surface boundary. Assuming that the coefficients of thermal conductivity and thermal diffusivity of the surface material are constant, one can obtain a solution for the temperature field in the form of a traveling wave which damps out within the depths of the wall and then the desired condition is

$$\Lambda\mu[c_p\vartheta'/\mathrm{Pr}+T_d\gamma'/\mathrm{Sc}]-\sqrt{a\alpha^2-i\alpha\,\mathrm{Re}C}\,\vartheta=0\ \text{when}\ y=0, \tag{1}$$

where $T_d = \varepsilon/kT_\infty$; $\Lambda = \sqrt{a}k\mu_\infty/2m\lambda^\circ$; $a = \lambda^\circ/\nu_\infty\rho^\circ c_p^\circ$; ε is the dissociation energy of a molecule; k is the Boltzmann constant; m is the atomic mass; ρ°, c_p°, and λ° are the density, specific heat capacity, and thermal conductivity of the surface material; $\nu_\infty = \mu_\infty/\rho_\infty$; c_p is scaled by k/2m.

The second condition follows from the balance between the number of atoms recombining on the surface and the number of atoms diffusing to the surface and has the form $\gamma'/c' = w^*/w$ when y = 0, where w and w* are the rate of surface recombination and its perturbation. The following assumptions are used with respect to recombination kinetics. The reaction is of first order, i.e., $w = k_0\rho(c - c_e)/(1 - c_e)$ when y = 0, where k_0 is a constant for the rate of recombination and c_e is the equilibrium degree of dissociation at the surface. The dependence of the equilibrium constant $K = 4pc_e^2/kT(1 - c_e^2)$ on temperature obeys the Arrhenius law $K = A\exp(-T_d/T)$. The relations for the equilibrium constant are used to eliminate pulsations of the equilibrium degree of dissociation in obtaining the final form of the boundary conditions:

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 40-45, January-February, 1978. Original article submitted February 7, 1977.

0021-8944/78/1901-0032$07.50

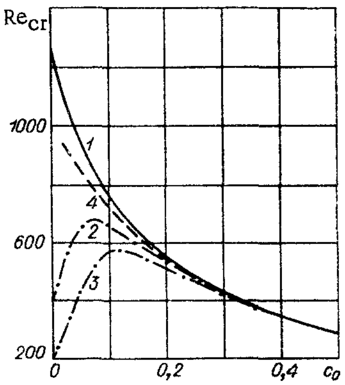

Fig. 1

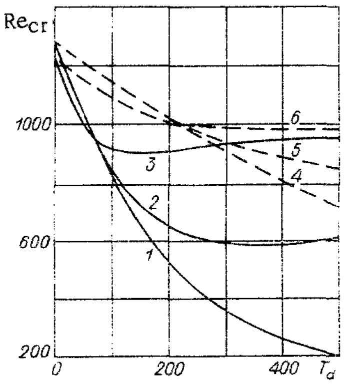

Fig. 2

$$c' \{(c - c_e + c_1)\pi - [c - c_e + c_1(1 + T_d/T)]\vartheta/T + (1 + c_e)\gamma/(1 + c)\} - (c - c_e)\gamma' = 0 \text{ when } y = 0,$$

where $c_1 = (1 - c)(1 + c_e)c_e/2$.

A gradientless boundary layer on a flat plate, which is not self-similar for a finite rate of recombination, is taken as the basic flow. Calculation of the boundary layer was performed using local similarity. This method yields exact results in the limiting cases of noncatalytic and ideally catalytic isothermal surfaces. The approximate results which are obtained with its help for a surface of finite catalytic activity are nearly exact [2] for a gradientless flow and are completely suitable for the studies made here.

A numerical calculation was performed for Pr = Sc = 1. It was assumed that the viscosity was independent of the degree of dissociation and was proportional to temperature and that the translational and rotational degrees of freedom of the molecule made the only contributions to the heat capacity. Only characteristic curves for neutral stability were calculated.

Figures 1 and 2 show calculated results for the critical Reynolds number $\mathrm{Re}_{cr} = \sqrt{u_\infty x_{cr}/\nu_\infty}$ for an isothermal boundary layer with $M_\infty = 0$ and $c_\infty = 0.5$. Curve 1 in Fig. 1, which shows the dependence of Re_{cr} on the degree of dissociation of the gas at a cold wall ($c_e = 0$) when $\Lambda = 0$, demonstrates that an increase in catalytic activity of the surface (i.e., a reduction in c_0) leads to an increase in stability. For a noncatalytic surface ($c_0 = c_\infty = 0.5$), as should be expected, the results agree with the results for a nondissociating gas. Cases of an ideally catalytic surface ($c_0 = c_e = 0$), where the flow is most stable, were studied at various values of T_0 and c_∞ in [3], where it was noted that Re_{cr} increases by more than an order of magnitude when c_∞ increases from 0 to 1.

The effect of heat release during recombination in the case of a cold wall is taken into account only by the second term in the square brackets in Eq. (1). Calculations show that the effect of the parameters a and Λ can be neglected when $a \lesssim 100$, $\Lambda \lesssim 0.01$. However, the parameter $T_D = \Lambda T_d$ can become important at large T_d. Curves 2 and 3 correspond to $T_D = 5$ and 10 and show that heat release resulting from the reaction leads to a reduction in Re_{cr} with it being most significant for a surface of high catalytic activity. It should be noted that an increase in the mean temperature of the surface because of the reaction was not considered here. It was assumed that all heat released by the reaction was removed and the effect of heat release on stability was only realized through the boundary conditions for a perturbation.

Curve 4 was plotted for $c_e = 10^{-2}$, $T_D = 5$, and $\Lambda = 0.1$ and shows that an increase in the equilibrium degree of dissociation (mainly because of heating) leads to a weakening of the effect of reaction exothermicity. The effect of c_e was considered in greater detail for an ideally catalytic surface ($c_0 = c_e$). Curves 1-3 in Fig. 2 correspond to $c_e = 0$, 10^{-3}, and 10^{-2} when $\Lambda = 0.02$ and curves 4-6 correspond to the same values of c_e when $\Lambda = 0.005$. It is clear that the reduction in Re_{cr} is limited for finite c_e so that the dependence on Λ and T_d becomes weak at large T_d. For intermediate values of T_d, all parameters are important including Λ; for example, Re_{cr} takes on smaller values when $\Lambda = 0.02$ than when $\Lambda = 0.005$. The cold-wall approximation is valid for small T_d and T_D is the dominant parameter. However, when c_e reaches values of the order of c_∞, the region for reduction of Re_{cr} is small and, consequently, the reduction itself is small. Therefore, stability is practically independent

TABLE 1.

Surface material	a	$\Lambda\cdot 10^3$	T_D
Metals	13	0,04	0,017
	0,51	0,23	0,1
Dense insulators	0,053	0,8	0,34
	0,002	4	1,7
Porous insulators	0,002	18	7,4

of heat release during recombination and the value of Re_{cr} can be determined from curve 1 in Fig. 1.

Typical values of the parameters a, Λ, and T_D for nitrogen when $T_\infty = 0$ (°C) and $p_\infty = 1$ atm are given in Table 1 for various surface materials. Nitrogen was chosen because it is a basic component of air and has a large dissociation energy corresponding to $T_d = 415$. A comparison of the tabulated data with the results obtained shows that the thermal effect of the reaction may be important under standard conditions only in the case of slightly thermally conducting and sufficiently catalytic walls (the catalyticity of which is provided, for example, by the deposition of a thin layer of catalyzer on the surface). All the results presented here were obtained under the assumption $a = 0$. Equation (1) indicates that this assumption is valid when $a \ll ReC/\alpha$; as already indicated, a direct calculation yields an estimate $a \lesssim 100$; the values given in Table 1 are much less. When the temperature and pressure differ from standard values, the approximation $\mu_\infty/T_\infty = \text{const}$ makes it possible to obtain the estimates

$$\Lambda \sim \sqrt{p_\infty},\ T_D \sim \sqrt{p_\infty}/T_\infty,\ a \sim p_\infty/T_\infty^2. \qquad (2)$$

It is obvious that for a metal wall these parameters can play a role only at high pressures (100 atm and above).

The instability region has a complex form for supersonic flows. Figures 3 and 4 present neutral curves with respect to $\omega = \alpha C/Re$ for $M_\infty = 5$. The solid curves correspond to an ideally catalytic surface, the dash–dot curves to a noncatalytic surface, and the dashed curves to a nondissociating gas. Calculations were made for $c_\infty = 0.5$, $c_e = T_D = \Lambda = 0$. It was assumed that the temperature of the surface was constant and equal to the temperature $T_0 = 6$ for a nondissociating gas at a thermally insulated surface. The results for a plate cooled to $T_0 = 3$, shown by curves 3-6 in Fig. 4, proved to be an exception. Curves 1 and 2 were obtained for three-dimensional perturbations propagating at an angle of 60°.

Curve 1 in Fig. 3 delimits the region of instability which is formed by combining the main region I, which also exists for subsonic flows, and the region II, which is the first of the infinite family of regions described in [4]. In addition, a previously unknown region III was found which extends to small Reynolds numbers corresponding to $\alpha Re \lesssim 1$ and which encompasses nearly the entire permissible range of C for given M_∞. For an ideally catalytic surface, this region joins with region II, which led to its discovery. The spatial extent of the instability region is small at large ω. One can therefore expect that the intensification factor for high-frequency perturbations is small and that the low frequencies typical of the main instability region I are of principal interest. It can then be seen that an increase in catalytic activity leads to a narrowing of the instability region as is also true for low flow rates. In contrast to the case $M_\infty = 0$, the results for a noncatalytic surface do not agree with the results for a nondissociating gas.

Figure 5 shows the dependence of Re_{cr} on c_0 for the main instability region when $M_\infty = 5$, $T_0 = 6$ and 3, and $M_\infty = 3$, $T_0 = 2.8$. The solid curves correspond to a reaction without thermal effect, the dash–dot curves to $T_D = 20$, $c_e = 0$, and the dashed curves to $c_e = 10^{-2}$, $T_D = 20$, $\Lambda = 0.01$. Comparison with the results for low flow rates (see Fig. 1) shows that the stability depends more weakly on both catalytic activity of the surface and heat release during recombination, particularly in the region of small c_0. A qualitative feature of the effect of heat release is that it increases flow stability. An increase in c_e, as in the case $M_\infty = 0$, weakens the effect of reaction exothermicity.

In the region of hypersonic flow, the results obtained can be of practical interest in the analysis of the stability of the boundary layer on the lateral surface of a blunt wedge or cone. Gas dissociated behind the bow shock rapidly expands, does not completely recombine,

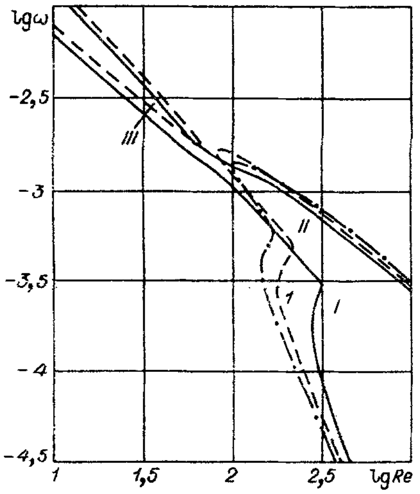

Fig. 3

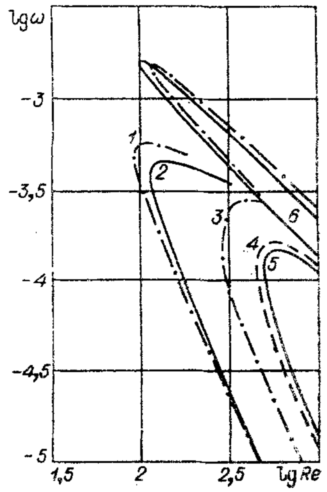

Fig. 4

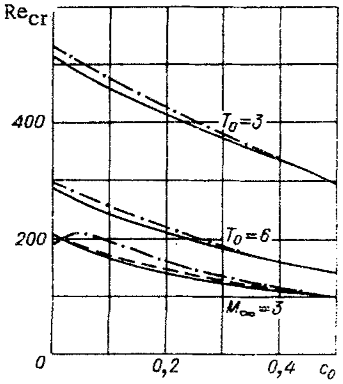

Fig. 5

and forms a stagnant and almost gradientless flow on the lateral surface [5]. Since the pressure in such flows is considerably less than 1 atm, the values in Table 1 and the estimates (2) indicate that the value of T_D are at least an order of magnitude below those used in the calculations. It becomes obvious that heat release exhibits practically no effect whatsoever through the boundary conditions for a perturbation and stability will be completely determined by the distribution of velocities and state parameters in the boundary layer.

LITERATURE CITED

1. D. W. Dunn and C. C. Lin, "On the stability of the laminar boundary layer in a compressible fluid," J. Aeronaut. Sci., 22, No. 7 (1955).
2. F. C. Blottner, "Chemical nonequilibrium boundary layer," AIAA J., 2, No. 2 (1964).
3. G. V. Petrov, "Stability of boundary layer of a gas with chemical reactions on a catalytic surface," Fiz. Goreniya Vzryva, 10, No. 6 (1974).
4. L. M. Mack, "Linear stability theory and the problem of supersonic boundary-layer transition," AIAA J., 13, No. 3 (1975).
5. L. I. Severinov, "Calculation of supersonic portion of perturbed region at a blunt body during supersonic nonequilibrium flow," Zh. Vychisl. Mat. Mat. Fiz., 8, No. 3 (1968).